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On the applicability of the load separation criterion to acrylonitrile/butadiene/styrene terpolymer resins

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Load separation constitutes the basis for the experimental evaluation of the J-integral by using the single-specimen technique. The objective of this present paper is to investigate the applicability of the load separation criterion for evaluating the ductile fracture mechanics parameters of acrylonitrile/butadiene/styrene (ABS) terpolymers. This criterion allows the load to be represented as the product of two separate functions, namely a material deformation function and a crack geometry function. Load separation implies a method for J-integral evaluation by using only a single load–displacement measurement. The original method for evaluating J used an energy-rate interpretation, which required several load–displacement measurements to be made for identical specimens with varying crack lengths. J methodology based on load separation introduces new parameters, i.e. η, η_{el} and η_{pl}, which greatly simplify the calculation of J and constitute the basis for the J–R multiple specimen technique. Recently, a method for both the experimental determination of the η-factors and the verification of load separation, which is based on separation constants, has been proposed for the testing of steel. This method allows the calibration of the η-factors in new test specimen geometries. This present paper attempts to evaluate experimentally whether the principle of load separation is valid when testing ABS polymers in a bending configuration in order to obtain a valid single J-testing method, and to calculate the η plastic factor experimentally by using this new simple method based on the load separation criterion in non-growing-crack measurements. In addition, a new approach based on the load separation principle which allows not only the calculation of J from a single specimen, but also the calculation of J–R curves from one test measurement has recently appeared. This approach is called the 'normalization method' and has already been applied to the J-testing of two rubber-toughened nylons and one ABS polymer. However, load separation has only demonstrated for different specimen geometries in steels. In addition, the load separation in ABS terpolymers has been verified in this present paper for growing cracks, by using an experimental procedure available in the literature. Copyright © 1996 Elsevier Science Ltd.

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INTRODUCTION

In 1968, Rice[1] proposed the J-integral as a new parameter for characterizing crack tip singularity in the elastic–plastic fracture behaviour of metals. This concept was subsequently applied to the fracture characterization[2-5] of various polymeric materials which displayed non-linear behaviour. The first investigations used an experimental technique to evaluate J which was based on the energy-rate interpretation of J as developed by Begley and Landes[6]. This requires the testing of many identically notched specimens with different crack lengths in order to establish the energy–crack length relationship from which J can be evaluated. Despite the reliability and the theoretical basis of this technique, it is not always a particularly practical method because of the need for large amounts of material for testing, and the time required for specimen preparation and testing, and also because of the lack of definition of the crack initiation point. A new technique that required the testing of only one specimen eventually succeeded the old technique and was thus widely accepted. This method is based on the assumption that the load can be represented as the product of two separate functions, namely a crack geometry function and a material deformation function. This separable form, which was first proposed by Rice *et al.*[7], brought a new definition of J, represented as a factor, defined later as η, multiplied by the area under the load displacement measurement, per unit uncracked ligament area. Hence, J can be evaluated by testing just one specimen if this factor is known for the specimen configuration. Based on the single-specimen technique, a new method[8] which is able to determine the crack-growth–material-resistance curve (the J–R curve) was developed. From the J–R curve an initiation toughness J_{IC}, can be determined.

This concept was very important in the development of standard test methods for metals[8], where all

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calculations of J are made by using the areas under load *versus* displacement curves, and it was rapidly extended to the evaluation of polymer toughness[9–16].

Recently, a new single-specimen technique known as the 'normalization method' has been developed[17] which enables the construction of a J–R curve from a single specimen without the need of an on-line monitoring system such as that used in the elastic compliance method in the ASTM E1152 J–R curve test standard. This normalization method also assumes load separation.

The values of η used in the current test methods, particularly for polymers, do not have a well established basis in analysis or experimental work. When the metals test standard was first published, the best values of η available at that time were incorporated[18]. No further development work was then carried out on η until some recent work in the field of metals testing[18–21].

Ernst and Paris[22] proved that η exists only if the load can be represented by a separable form. This is true by definition for elastic behaviour because of the linearity of the load–displacement measurements, as will be explained below. As there are many linear-elastic solutions already available, it is not generally difficult to determine η_{el}. However, for the plastic factor there continues to be much controversy and it is not easy to calculate this factor for some structures because of the non-linearity[23].

Despite the fact that load separation has been analytically demonstrated for Ramberg Osgood materials by the EPRI handbook solutions[24], it has only been experimentally investigated for a very few number of configurations and materials[20,25]. Moreover, a slight material sensitivity has been reported[18].

In the following, load *versus* displacement measurements will be used to evaluate the load separation validity in ABS resins testing in the bending configuration by using the experimental procedure proposed by Sharobeam and Landes[20]. The results of this evaluation will be used to assess the accuracy of the η_{pl} plastic factor (normally equal to 2) used in the J testing of polymers in the bending configuration[8–16], as theoretically derived by Rice *et al.*[7]. In addition, the extension of this load separation approach to growing crack measurements on the same ABS polymers was investigated by following a modification of the procedure proposed by Sharobeam and Landes[19].

THE J-INTEGRAL

J may be defined as the energy per unit area necessary to create new surfaces. At crack initiation, it may be determined from considering the load–deflection curves of two bodies with crack lengths of a and $a + \mathrm{d}a$. This may be expressed as follows:

$$J = -\frac{1}{b}\left.\frac{\partial U}{\partial a}\right|_{\nu} \tag{1}$$

where ν is the displacement, U is the potential energy of the loaded body (the energy given by the area under the load–deflection curve), b is the uncracked ligament length, and $J = J_c$ at fracture. This energy definition of the J-integral was proposed as a fracture criterion for the elastic–plastic behaviour of metals and extended the linear elastic fracture mechanics (LEFM) concepts to cases in which large scale plasticity is envolved.

J may be experimentally determined by taking a set of specimens of different crack lengths and measuring the load–deflection curves. From these U *versus* crack length curves may be constructed at various constant displacements and thus J can be found for any crack length and displacement from the slopes of the lines[2–5]. However, the multiple-specimen test procedure[6] is not only time consuming but also requires a large amount of material for test specimens.

The single-specimen technique was first developed by Rice *et al.*[7] for deeply cracked bend specimens and by Merkle and Corten for compact specimens[26]. Both analyses are based on load separable forms developed by using limit load analysis[20].

Under the above conditions, Rice *et al.*[7] obtained an expression for J as follows:

$$J = \frac{2}{b}\int M\,\mathrm{d}\theta \tag{2}$$

where θ is the additional rotation due to the presence of the crack, M the bending moment and ν the displacement.

The use of a separable form to represent the load created a new factor, η, which relates the work done per unit pre-cracked ligament area in the loading of a cracked body to the J-integral. For deeply notched specimens under bending, η is equal to 2.

This method, based on load separation, simplifies the J calculation and constitutes the basis for the multiple-specimen technique for J–R curve determination[8].

LOAD SEPARATION ANALYSIS

Load separation is the assumption that the load in the test measurements of specimens of the same material, geometry and constraint can be represented as a product of two separate functions, namely a crack geometry function and a material deformation function[19]. This can be expressed mathematically as follows:

$$P(a,\nu) = G(a)H(\nu) \tag{3}$$

Sumper and Turner[27] proposed the following expression for J:

$$J = \eta_{el}\frac{A_{el}}{b} + \eta_{pl}\frac{A_{pl}}{b} \tag{4}$$

where A_{el} and A_{pl} are the elastic and plastic parts of the area under the load displacement measurement, respectively, and η_{el} and η_{pl} are the functions of the crack length to width ratio, a/W. They used LEFM relationships to determine η_{el}, but η_{pl} was calculated by using numerical methods. This new form marked the first generalization of the modification factor as η, and assumed a separation into both elastic and plastic behaviours. For the elastic region, this was proved experimentally and analytically because the LEFM approach assumes the linearity of the load–displacement measurements, as follows:

$$P = \frac{\nu}{C} \tag{5}$$

where C is the specimen compliance, which is a function of the crack length only. Because equation (5) represents a separable form, η_{el} can easily be shown to be represented by the following expression:

$$\eta_{el} = \frac{b}{C}\frac{dC}{da} \tag{6}$$

Ernst and Paris[22] discussed the separability for J and P in the plastic region and suggested different separable forms for P. They also investigated analytically the validity of these forms and the associated η_{pl} factors for bending and tension configurations. They proved that η exists only if the load is represented by a separable form. According to Paris *et al.*[28], the single-specimen J-form and the η factor exist only if the load is separable.

It is also worth noting here that the metal standard test methods for J_{IC} (plain strain fracture toughness in mode I) and the J–R curves[8,9] use the following form to represent J:

$$J = \frac{K^2}{E'} + \eta_{pl}\frac{A_{pl}}{b} \tag{7}$$

where K is the stress intensity factor, $n_{pl} = 2$ for bend specimens, and $2 + 0.522\ b/W$ for compact tension specimens.

This form, which has also been adopted by the European Protocol[29], assumes load separation into both elastic and plastic regions.

Load separation in stationary cracks

Sharobeam and Landes[20] stated that if the load, for a given material, geometry and constraint, could be represented in a separable form, as follows:

$$P = G(a/W)H(\nu_{pl}/W) \tag{8}$$

then, for two test measurements of the load–plastic displacement of different stationary crack lengths a_i and a_j, a parameter S_{ij}, defined as $P(a_i)/P(a_j)$ at constant ν_{pl}, will have a constant value over the whole domain of the plastic displacement, as a result of the following relationships:

$$S_{ij} = \left.\frac{P(a_i)}{P(a_j)}\right|_{\nu_{pl}} \tag{9}$$

and

$$S_{ij} = \frac{G(a_i/W)}{G(a_j/W)} \tag{10}$$

As the geometry function is constant for stationary cracks, then equations (9) and (10) imply that the separation parameter, S_{ij}, is a constant for fixed values of a_i and a_j and is not a function of ν_{pl}. The constancy of S_{ij} implies that over the whole domain of ν_{pl} the load can be represented by a separable form (*Figure 1*).

By evaluating $P(a_i)/P(a_j)$ we can determine, for a given material, geometry and constraint, if the load is separable and the ν_{pl} range of separability.

η plastic factor calculations

In investigations[30,31] made prior to the work of Sharobeam and Landes[20], η_{pl} was experimentally evaluated by comparing the results of the two forms of J_{pl}, i.e. the energy-rate interpretation form with the single-specimen form. This method is very complicated and generates errors arising from area and slope determinations[18]. Using the separable form (equation (8)), Sharobeam and Landes[20] derived an alternative analytical form for η_{pl}, as follows:

$$\eta_{pl} = \frac{dG(b/W)/d(b/W)}{G(b/W)}\frac{b}{W} \tag{11}$$

Equation (11) shows the relationship between the geometry function $G(b/W)$ and the η_{pl}-factor. The $G(b/W)$ function can be constructed from the experimental data by using the separation constants S_{ij} for the different measurements as follows:

$$S_{ij} = C_1.G(b_j/W), \text{ for constant } b_j/W \tag{12}$$

where C_1 is a constant equal to $1/G(b_j/W)$.

This means that by constructing the S_{ij} vs. a_i/W (or b_i/W) plot, an analytical $G(b_i/W)$–b_i/W relationship could be eventually established; η_{pl} can then be simply evaluated from equation[12]. When a power law fits the geometry function accurately, as in the case of certain metals[20], this new method provides a very simple experimental way of calculating η_{pl}, as follows:

$$G(b_j/W) = C_2(b_j/W)^m \tag{13}$$

where C_2 is a constant.

By substituting equation (13) into equation (11), we then obtain:

$$\eta_{pl} = m \tag{14}$$

Load separation in growing cracks

Sharobeam and Landes[19] proved that load separation can also be extended to growing cracks. They again represented the load for the same material, geometry and constraint by the following:

$$P = G_p(b_p/W)H_p(\nu_{pl}/W) \tag{15}$$

where $C_p(b_p/W)$ and $H_p(\nu_{pl}/W)$ are the proposed geometry and deformation functions, respectively, for the

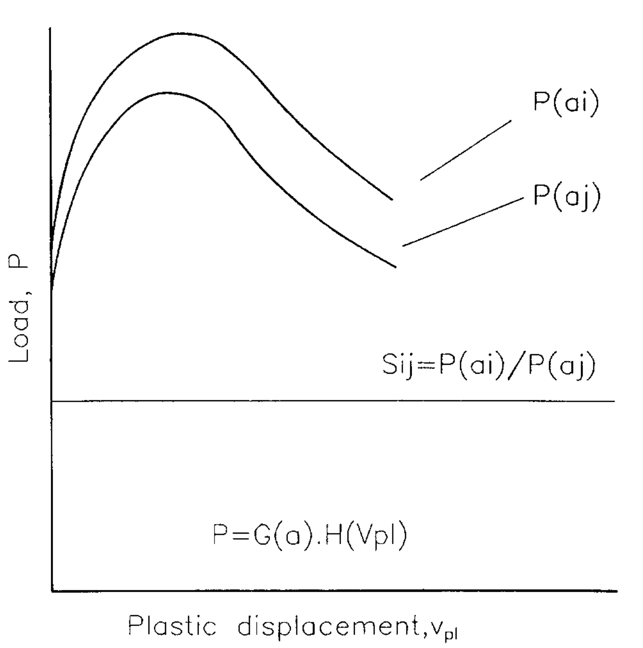

Figure 1 Load separation criterion in the plastic region: load vs. plastic displacement measurements for two stationary cracks a_i, and a_j, and the corresponding separation parameter, S_{ij}

pre-cracked specimen test measurements which are assumed to be different to those previously proposed for the stationary crack test measurements. They normalized the load by the separable form proposed for the stationary crack specimen, and by assuming consistent material, geometry and constraints, obtained the following expression:

$$\frac{P_\mathrm{p}}{P_\mathrm{b}} = \frac{G_\mathrm{p}(b_\mathrm{p}/W)H_\mathrm{p}(\nu_\mathrm{pl}/W)}{G_\mathrm{b}(b_\mathrm{b}/W)H_\mathrm{b}(\nu_\mathrm{pl}/W)} \tag{16}$$

where the subscripts p and b denote pre-cracked and blunt notched (or stationary crack) specimens, respectively. In the stationary crack test measurement, $G_\mathrm{b}(b_\mathrm{b}/W)$ is a constant.

Based on the above considerations, they were able to define a separation parameter, S_pb, as follows:

$$S_\mathrm{pb} = \left.\frac{P_\mathrm{p}}{P_\mathrm{b}}\right|_{\nu_\mathrm{pl}} \tag{17}$$

$$S_\mathrm{pb} = C_3 G_\mathrm{p}(b_\mathrm{p}/W)H_\mathrm{pb}(\nu_\mathrm{pl}/W) \tag{18}$$

where C_3 is a constant equal to $1/G_\mathrm{b}$ and $H_\mathrm{pb}(\nu_\mathrm{pl}/W)$ is the ratio of the deformation functions at a constant plastic displacement: this can be evaluated for the different growing crack test measurements with respect to the same stationary crack test measurement as follows:

$$S^\mathrm{i}_\mathrm{pb} = C_3 G^\mathrm{i}(b_\mathrm{p}/W)H^\mathrm{i}_\mathrm{pb}(\nu_\mathrm{pl}/W) \tag{19}$$

where the superscript i denotes the pre-cracked specimen. Different pre-cracked specimen results were plotted together on one graph in the form of S^i_pb vs. b_p/W, proving that S_pb could be considered as being a function of b_p/W only, and that the plastic displacement has no contribution to the separation parameter, thus demonstrating that load separation for the growing crack measurements can be represented by the following expression:

$$S_\mathrm{pb} = C_3 G(b_\mathrm{p}/W) \tag{20}$$

It was found that the same geometry function that best-fitted the blunt notched experiments was also well fitted to pre-cracked load separation experiments.

EXPERIMENTAL

Materials and sample preparation

A commercial grade material has been investigated, namely an ABS-type resin (Lustran ABS 850), which was kindly provided by Unistar Argentina SA. Pellets of the ABS resin were dried at 85°C for 2 h under vacuum and then compression moulded at 195°C into plates of 7 mm thickness (B).

In order to release the residual stresses generated during moulding, all of the plaques were submitted to a post-moulding thermal treatment consisting of keeping the samples for 1 h at 120°C under a slight pressure, and then slowly cooling to room temperature within the oven.

Non-growing-crack experiments were conducted on U-notched three-point bend specimens which were cut from the compression moulded plates. U-notches were introduced into the samples by machining to give crack-to-depth ratios (a/W) varying between 0.4 and 0.7. The thickness-to-depth ration (B/W) and span-to-depth ratio (S/W) were always kept equal to 0.5 and 4 respectively. Blunt notches were used in order to retard crack initiation up to sufficiently large displacements.

Testing procedure and data handling

Mechanical testing was carried out at room temperature, at a crosshead rate of $2\,\mathrm{mm\,min^{-1}}$, in a Shimadzu Autograph S-500-C Universal testing machine.

The load signal was digitized by a board in an AT class computer; the data sample interval was $\Delta t = 1$ s. Load displacement data were stored via a 16-bit A/D converter in the AT class computer memory and were thus available for subsequent analysis.

A series of identical specimens differing in their stationary crack lengths were tested, and the load-deflection curves were determined.

The actual compliances were calculated from the initial slopes of the load–displacement curves and the elastic displacements were subtracted from the total displacement. The plastic displacements were obtained as follows:

$$\nu_\mathrm{pl} = \nu - CP \tag{21}$$

Then, by subtracting the elastic displacement, a new load *versus* plastic displacement reading was determined. The $a/W = 0.55$ measurement was taken as the reference. Subsequently, the separation parameters, S_{ij}, were evaluated for each specimen test, by dividing the load by the reference at different values of the plastic displacement.

From a plot of S_{ij} *versus* ν_pl, the range in which the separation parameter remains constant was determined, and as a consequence, the existence of the load separability is guaranteed. Within this range, S_{ij} *versus* b_i/W (uncracked ligament to width ratio) lines were constructed at several arbitrary plastic displacement values in order to evaluate η_pl. *Figure 2* shows a schematic representation of the testing procedure that was adopted.

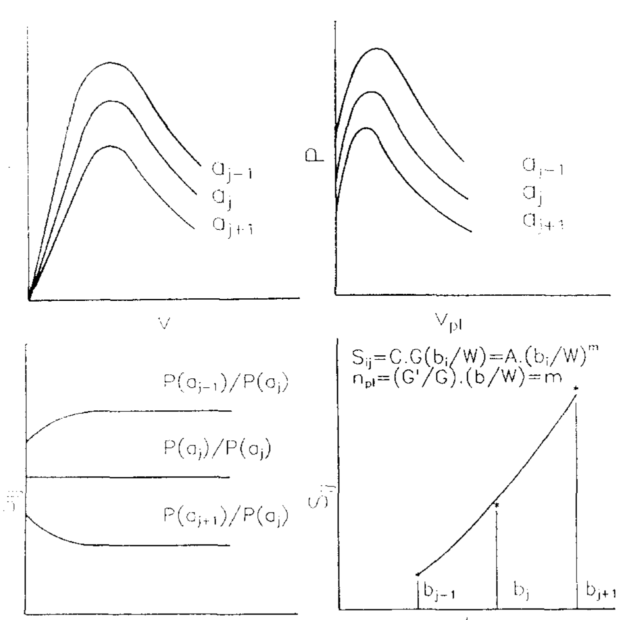

Figure 2 Flow chart illustrating the η_pl calculation methodology

RESULTS AND DISCUSSION

Figure 3 shows the original load–displacement traces for blunt notched specimens at different a/W ratios, and *Figure 4* shows the new load–plastic displacement plots obtained from the original load–displacement measurements after subtracting the elastic displacement. The load–plastic displacement plots are somewhat different from those found for metals[18,20,21]. The load shows an increasing trend with plastic displacement up to a maximum, at which point the load began to drop. The load relaxation suggests that the crack has already propagated, and therefore the assumption of a non-growing-crack regime is not still valid.

Figure 5 shows the variation of the separation parameter with respect to the plastic displacement for ABS samples at different a/W ratios, calculated from the load–plastic displacement results as explained above.

The separation parameters maintain an almost constant value over a sufficient part of the plastic displacement range, except for a limited region at the beginning of plastic behaviour, i.e. ν_{pl} less than 0.11 and larger than 1.08 mm. The latter indicates that within the plastic interval being considered the load in the ABS specimens is separable and can be represented as a product of two separate functions (as proposed by equation (8)).

As emerges from *Figure 5*, there is a unseparable region at the early plastic behaviour, as seen from the non-constancy of the separation parameter. This kind of behaviour was also observed in steels[18], and implies that within that region the expression $J_{pl} = \eta_{pl}A_{pl}/b$ is not valid, and hence η_{pl} does not exist. The inseparable behaviour may be associated with the transition from elastic to plastic behaviour where small-scale yielding is generally observed.

The separation parameter could also be used to check the validity of the stationary-crack results. S_{ij} starts to drop, at a certain level of ν_{pl} (1.08 mm), coincident with incipient load relaxation (*Figure 4*), due to the initiation of crack propagation.

In order to develop the geometry function, data obtained from different experiments at different plastic displacement levels, within the region where S_{ij} maintains its constancy, were plotted together. *Figure 6* shows the separation parameter S_{ij} *versus* b_i/W. The data reduce into one curve, demonstrating that the geometry function is not dependent on the plastic displacement. It is obvious that a power-law function will accurately fit these data. From equation (17) the calculation of η_{pl} is now very simple, and gives a result of 2.0014. This value may be considered as being essentially equal to the traditional value of 2 derived theoretically by Rice *et al.*[7] by limit-load analysis and widely adopted in the literature for three-point-bending specimens.

Additional calculations based on the recent investigations made by Sharobeam and Landes[19] regarding load

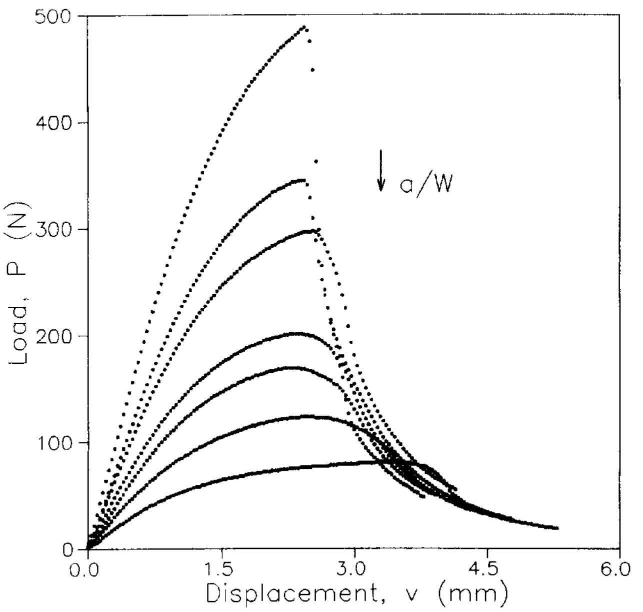

Figure 3 Load vs. total displacement plots for ABS blunt notched specimens with different notch lengths

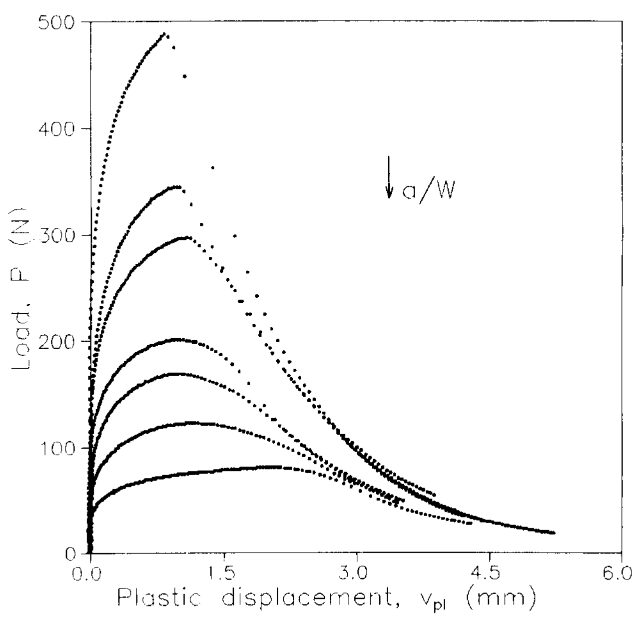

Figure 4 Load–plastic displacement data for ABS blunt notched specimens with different notch lengths

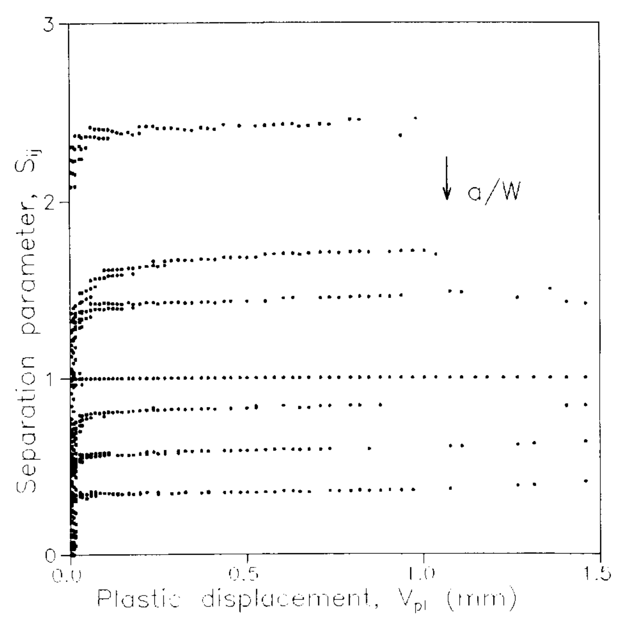

Figure 5 The separation parameter S_{ij} vs. plastic displacement ν_{pl}; reference crack, $a_j/W = 0.55$

separation in growing-crack measurements were carried out by using information published elsewhere[13] obtained from multiple-specimen experiments. In order to account for the difference in specimen widths between blunt notched and pre-cracked specimens, a slight modification was introduced in the S_{pb} definition (equation (17)). From the comparison between the deformation function and the normalized load[32,33] it emerges that:

$$S_{pb} = \frac{P_p}{P_b}\bigg|_{\nu_{pl}} \frac{W_b^2}{W_p^2} \tag{22}$$

The separation parameter for growing-crack measurements was calculated for pre-cracked specimens

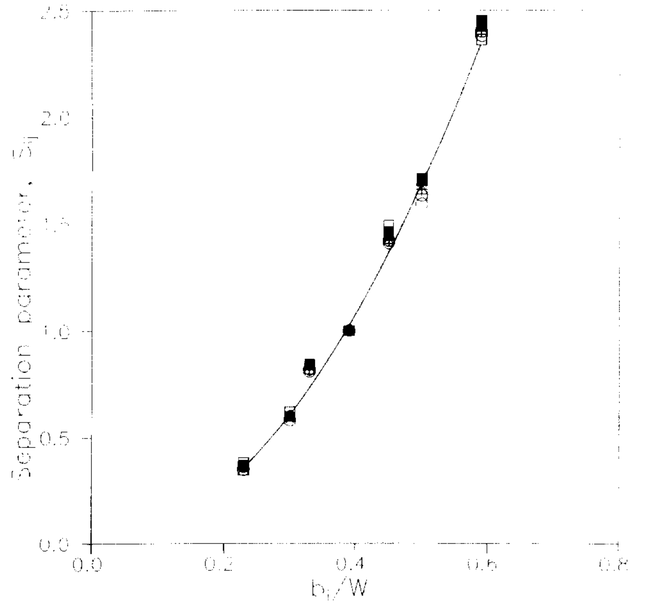

Figure 6 The separation parameter S_{ij} vs. uncracked ligament to width ratio for different values of the plastic displacement: (○) 0.11 and 0.20; (△) 0.29; (+) 0.41; (×) 0.55; (●) 0.63; (▲) 0.73; (■) 0.82; (□) 0.94 and 1.08 mm

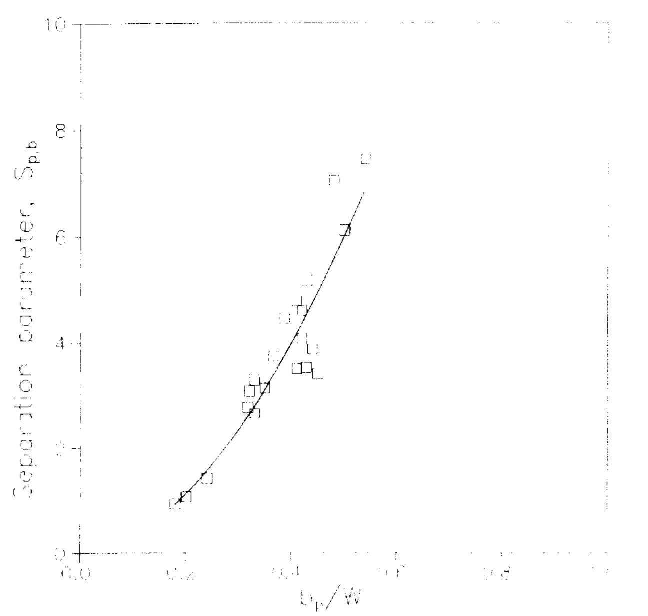

Figure 7 Separation parameter vs. uncracked ligament to width ratio using data obtained from growing-crack measurements; $W_b = 14$ mm, $10\,\mathrm{mm} < W_p < 20\,\mathrm{mm}$, $S_{pb} = 21.15(b_p/W)^{1.82}$

advanced to different a_i levels. *Figure 7* shows the corresponding results. The data can be fitted together by a power-law function which demonstrates load separation in these growing-crack experiments.

A theoretical S_{pb} expression (equation (20)) was also calculated by considering that C_3 was equal to the inverse of the geometry function of the blunt notched specimen and that the geometry function in the pre-cracked specimen is the theoretical one, leading to $S_{pb} = 18.58\,(b_p/W)^2$. The differences between the fitted and theoretical expressions may be due to experimental errors derived from the multiple-specimen technique that is used, such as crack-growth determination, and/or the assumption of a non-growing crack in the blunt specimen experiment not being completely true.

Our results show that the load separation principle can also be extended to growing-crack measurements up to sufficiently large plastic displacement levels, thus allowing cracks to grow up to more than 30% of the uncracked ligament length, which represent deeper crack advances than those allowed by current standard exclusion lines[29]. Hence the J–R curves could be determined with greater confidence.

CONCLUSIONS

The load separation hypothesis has been experimentally proven for an ABS polymer over a sufficiently wide range of plastic displacement under static conditions for stationary and non-stationary cracks in the bending configuration by using the Sharobeam and Landes methods[19,20]. The method is simple and easy to apply. This finding implies that the use of a single-load displacement measurement method for the ABS J-integral evaluation is viable, and therefore the normalization method could also be used in order to evaluate J–R curves[32] from a single test measurement.

The testing protocol exclusion lines[29] only allow crack-growth levels up to 10% of the uncracked ligament length. The growing-crack results suggest that the protocol exclusion lines are too conservative for testing ABS polymers.

The geometry function has been developed as a power-law function. Its exponent, which is coincident with the η_{pl} value, is found to be very close to 2, which is the value theoretically derived by Rice *et al.*[7] and widely adopted in the J-testing of polymers in the literature[9–16].

In addition, this technique also represents a powerful tool for the experimental calibration of η_{pl} for the J-testing of polymers in new geometries. Further work is in progress regarding this latter issue. Additional work has also been focused on generalizing these findings to other polymers[33] which display different yield behaviour.

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NOMENCLATURE

a	crack length
A	area under the load–displacement measurement
b	uncracked ligament length
b	as subscript, refers to blunt notched specimen
C	elastic compliance
el, pl	as subscripts, refer to elastic and plastic components respectively
E	Young's modulus
E'	$E/(l-\nu^2)$
G	geometry function
H	material deformation function
H_{pb}	ratio of the deformation functions for pre-cracked and blunt notched specimens
J	J-integral
J_{IC}	plane strain fracture toughness
K	stress intensity factor
M	bending moment
P	load
p	as subscript, refers to pre-cracked specimen
S	separation parameter
U	potential energy of the loaded body
W	specimen width

Greek symbols

η	eta-factor
θ	additional rotation due to presence of the crack
ν	displacement
ν	Poisson ratio